

# Analog model for quantum gravity effects: phonons in random fluids

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We describe an analog model for quantum gravity effects in condensed matter physics. The situation discussed is that of phonons propagating in a fluid with a random velocity wave equation. We consider that there are random fluctuations in the reciprocal of the bulk modulus of the system and study free phonons in the presence of Gaussian colored noise with zero mean. We show that in this model, after performing the random averages over the noise function a free conventional scalar quantum field theory describing free phonons becomes a self-interacting model.

Unruh has shown that the propagation of sound waves in a hypersonic fluid is equivalent to the propagation of scalar waves in a black hole spacetime [1]. Quantizing the acoustic wave in such physical system with sonic horizon implies that the sonic black hole can emit sound waves with a thermal spectrum, recovering the Hawking result [2] in this analog model, i.e. the presence of phononic Hawking radiation from the acoustic horizon. Since the Unruh fundamental paper, the possibility of simulating aspects of general relativity and quantum fields in curved spacetime through analog models has been widely discussed in the literature [3, 4]. In the context of quantum gravity, most of the work on analogue models has focussed attention on violation of Lorentz invariance. The key point of the discussion is the expectation of a low-energy manifestation of a possible spacetime discreteness at the Planck scale. Such a manifestation is precisely violation of Lorentz invariance. In these analog models interesting dispersion relations are derived [5, 6].

The aim of the present paper is to take a somewhat different look at the way an analog model for quantum gravity effects can be implemented. Here we have in mind the results obtained by Ford and collaborators [7], where fluctuations of the geometry of spacetime caused by a bath of squeezed gravitons has the effect of smearing out the light cone. In addition to quantum mechanical metric fluctuations, there are induced metric fluctuations generated by quantum fluctuations of matter fields. In the regime where induced metric fluctuations dominate, the conventional approach is to assume a stochastic ensemble of fluctuating geometries. In this framework, Hu and Shiokawa [8] assumed that the metric tensor has a deterministic and a stochastic contribution [9]. We argue that in condensed matter physics there is an analog model for fluctuations on the light cone generated by a thermal bath of squeezed gravitons. We consider a disordered medium with random classical fluctuations in the recip-

rocal of the bulk modulus. Actually, the analog model we discuss in the paper is closely related to the induced metric fluctuations framework. We consider a scalar quantum field theory associated to acoustic waves defined in a fluid with classical random fluctuations, where the velocity of propagation of the acoustic waves fluctuates randomly, and consequently the sonic cone fluctuates.

Fluctuations of a fluid consist of two types, classical and quantum. Quantum fluctuations are due to quantization of fluid thermodynamic observables. Consideration of quantum fluctuations motivated Ford and Svaiter [10] to discuss the scattering of light by the zero-point density fluctuations of a phonon field. Also, since there is an analog to the Casimir effect when the zero-point fluctuations of the phonon field are submitted to boundary conditions [11], the same authors considered in Ref. [12] the effects of boundaries discussing the density correlation function associated to the phonon field in many different situations. In the present paper we will show that fluids are not only useful analog models for effects in field theory, but random fluids with particular properties can simulate quantum gravity effects.

Let us start from the basic equations of fluid dynamics [13]. The local mass density  $\rho(t, \vec{r})$ , the pressure  $p(t, \vec{r})$ , the local velocity  $\vec{v}(t, \vec{r})$  and the local temperature  $T(t, \vec{r})$  are the thermodynamic fields of the system. For an ideal fluid, assuming thermodynamical equilibrium, the acoustic wave equations are obtained by linearizing the fluid dynamics equations for small disturbances around the fluid at rest, namely

$$p(t, \vec{r}) = p_0 + \delta p(t, \vec{r}), \quad (1)$$

$$\rho(t, \vec{r}) = \rho_0 + \delta \rho(t, \vec{r}), \quad (2)$$

$$\vec{v}(t, \vec{r}) = \delta \vec{v}(t, \vec{r}), \quad (3)$$

where  $\rho_0$  and  $p_0$  are the constant equilibrium density and pressure. Using the Euler and mass balance equations, one obtains

$$\frac{\partial}{\partial t} \delta \vec{v}(t, \vec{r}) + \frac{1}{\rho_0} \nabla \delta p(t, \vec{r}) = 0, \quad (4)$$

$$\frac{\partial}{\partial t} \delta \rho(t, \vec{r}) + \rho_0 \nabla \cdot \delta \vec{v}(t, \vec{r}) = 0. \quad (5)$$

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The bulk modulus  $K$  of the fluid is defined as  $K = \rho_0 \partial p / \partial \rho = \rho_0 u^2$ , where  $u = (\partial p / \partial \rho_0)^{1/2}$  is the sound velocity. Using that the sound wave in a ideal fluid is adiabatic,  $\delta p(t, \vec{r}) / \delta \rho(t, \vec{r}) = \partial p(t, \vec{r}) / \partial \rho_0$ , one obtains from the fluid equations a wave equation for  $\psi(t, \vec{r}) \equiv \delta p(t, \vec{r})$

$$\left( \frac{1}{u^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \psi(t, \vec{r}) = 0. \quad (6)$$

Here, in general  $u$  can depend on  $\vec{r}$ . This wave equation, as can be easily checked, is similar as the wave equation one would obtain if considering a curved spacetime where the metric contains a deterministic and a stochastic contribution, similar to the situation discussed in Ref. [8], where metric fluctuations were represented as fluctuations in the optical index associated to electromagnetic waves. This equation also appears in the study of elementary excitations in disordered elastic media with random fluctuations, which has been discussed recently by many authors. For example, it has been pointed out that a free phonon field in a disordered elastic medium becomes self-interacting [14]. As we will show in the following, a similar situation occurs when  $u(\vec{r})$  fluctuates.

When  $u$  is a constant, one can derive a wave equation also for  $\delta \rho(t, \vec{r})$  and  $\delta \phi(t, \vec{r})$ , with  $\delta \vec{v} = \nabla \delta \phi$ . In this case the quantization of the system follows as usual [15], the phonons have a linear dispersion relation  $\omega(\vec{k}) = u |\vec{k}|$  and the operators  $\delta \rho(t, \vec{r})$  and  $\delta \phi(t, \vec{r})$  obey equal-time commutation relation given by [we assume  $\hbar = 1$ ]

$$[\delta \phi(t, \vec{r}), \delta \rho(t, \vec{r}')] = -i \delta^3(\vec{r} - \vec{r}'). \quad (7)$$

The unperturbed two-point Green's function of the phonon field  $D_F(x, x')$  is given by the expectation value of the time ordered product

$$D_F(x, x') = -i \langle 0 | T(\delta \rho(t, \vec{r}) \delta \rho(t', \vec{r}')) | 0 \rangle, \quad (8)$$

where  $|0\rangle$  is the ground state of the phonon field.

Having performed a linearization to obtain a wave equation for the acoustic perturbation, let us consider the influence of random fluctuations over the phonon field. Under such circumstances, the fluid equations become random differential equations. The simplest assumption one can make is to consider a quantized phonon field in a fluid with bulk modulus that fluctuates randomly, with  $\rho_0$  kept constant [16]. This generates fluctuations in the velocity of the acoustic perturbations. Therefore, let us assume (for simplicity we shall consider one-dimensional fluids, the extension to three dimensions is not difficult)

$$\frac{1}{u^2(z)} = \frac{1}{u_0^2} (1 + \nu(z)), \quad (9)$$

with  $u_0$  the sound speed in a homogenous medium and  $\nu(z)$  is taken to be a Gaussian colored noise with zero mean

$$\langle \nu(z) \rangle_\nu = 0, \quad \langle \nu(z) \nu(z') \rangle_\nu = \sigma^2 h_\Lambda(z - z'). \quad (10)$$

The symbol  $\langle \dots \rangle_\nu$  denotes average over noise realizations. Under such circumstance, the fluid equations become random differential equations and two important points should be noted. First, one needs to specify a stochastic modeling of the fluctuations of the medium parameters in terms of random processes. Second, the scale separation in the problem should be properly addressed. This second point will be discussed latter ahead in the paper. With respect to the first point, we mention that the specific form of  $h_\Lambda$  is not relevant for our arguments, it is sufficient to note that  $\Lambda \sim 1/l_d$ , where  $l_d$  is the correlation length of the noise field, and for  $\Lambda \rightarrow \infty$  one has uncorrelated white noise. However, it should be clear that  $\nu(z)$  is modeling fluctuations of a spacetime background on which a quantum field is propagating and it should be related to geometry fluctuations on a more fundamental level. For example, two-dimensional geometries are characterized by the scalar curvature  $R$  of the Riemann tensor and one could speculate that  $\nu(z)$  should be related to  $R$ . We will not try to make such a connection here because this would take us away from the main scope of the paper.

Our aim is to calculate the density-density correlation function, which is related to an observable as e.g. in a light (or neutron) scattering experiment. This is most easily calculated solving first Eq. (6) and then using the relation  $\psi(t, z) = u^2(z) \delta \rho(t, z)$ . We solve Eq. (6) by a conventional perturbation expansion in  $\nu(z)$ . We write  $\psi(t, z) = \sum_{n=0}^{\infty} \psi_n(t, z)$ , where  $\psi_n(t, z)$  is of order  $n$  in  $\nu(z)$ . Substituting this in Eq. (6) and collecting terms of the same order in  $\nu(z)$ , one obtains the recursive relation for  $n > 0$

$$\left( \frac{1}{u_0^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right) \psi_n(t, z) = -\frac{\nu(z)}{u_0^2} \frac{\partial^2}{\partial t^2} \psi_{n-1}(t, z). \quad (11)$$

For  $n = 0$ , we have the standard wave equation for  $\psi_0(t, z)$ . Choosing the solution of the homogeneous equation to be zero, the general solution of Eq. (11) can be written as

$$\psi_n(t, z) = \int dz' dt' G_F(t - t'; z - z') \frac{\nu(z')}{u_0^2} \frac{\partial^2}{\partial t'^2} \psi_{n-1}(t', z'), \quad (12)$$

where  $G_F(t - t'; z - z')$  is the Green's function associated with the differential operator on the l.h.s. of Eq. (11). Introducing a Fourier representation for  $\psi_n(t, z)$

$$\psi_n(t, z) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \psi_n(\omega, z), \quad (13)$$

and a similar one for  $G_F(t - t'; z - z')$ , the complete solution can be written as

$$\psi(\omega, z) = \psi_0(\omega, z) + \sum_{n=1}^{\infty} (-1)^n \left( \frac{\omega}{u_0} \right)^{2n} \int \prod_{m=1}^n dz_m G_F(\omega; z_{m-1} - z_m) \nu(z_m) \psi_0(\omega, z_n), \quad (14)$$

where it is to be understood that  $z_0 = z$ .

Given the solution, one can calculate the density-density correlation function using  $\delta\rho(t, z) = \psi(t, z)/u^2(z)$  and quantize as usual. It is important to call attention to the validity of the linear dispersion relation in the case of randomness. This situation is similar to the relation of the light cone fluctuations to Lorentz symmetry. In the same way as light cone fluctuations respect Lorentz symmetry on the average, the linear dispersion relation leads to  $k^2/\omega^2 = (1 + \nu(z)) k_0^2/\omega_0^2$ , which should be considered as valid on the average.

Before proceeding, let us abstract for the moment from the fact that  $\psi$  represents the pressure field of the fluid and consider it as a scalar field *per se*. Eq. (6) represents a wave equation for a free scalar field in a fluctuating spacetime metric. Suppose one quantizes the field  $\psi$  and calculate the two-point correlation function by taking the statistical and quantum averages  $\langle \langle \dots \rangle_\nu \rangle$ , i.e. average over noise realizations and vacuum expectation value of products of field operators. The two-point correlation function up to second order in  $\nu(z)$  would be given by (in terms of the Fourier transforms of  $G_F$  and  $h_\Lambda$ )

$$\langle \langle \psi(\omega, k) \psi(\omega, k') \rangle_\nu \rangle = (2\pi)^2 \delta(\omega - \omega') \delta(k - k') S_\psi(\omega, k), \quad (15)$$

where

$$S_\psi(\omega, k) = iG_F(\omega, k) + iG_F(\omega, k) \Pi(\omega, k) iG_F(\omega, k), \quad (16)$$

with

$$\Pi(\omega, k) = -2\sigma^2 \frac{\omega^4}{u_0^4} \int dq iG_F(\omega, q) h_\Lambda(k - q), \quad (17)$$

where we used the fact that  $\langle 0|T(\psi_0(t, z)\psi_0(t', z'))|0\rangle = iG_F(t - t', z - z')$ .

It is clear that Eq. (17) leads to the interpretation that a non-interacting scalar field propagating in a fluctuating metric turns it into a self-interacting field. Notice

that our results are valid only if the background fluctuations are much larger than those fluctuations defined in equations (1)-(3). Nevertheless, the scale separation in the problem mentioned above can be analyzed in terms of its low- and high-frequency regimes. We notice that the contributions of the perturbations to the two-point (and  $n$ -point) functions have a dependence on  $\omega$  in a multiplicative way. More precisely, the deviation from a uniform medium occurs multiplicatively with frequency. The consequences of this observation are clear; for large frequencies, the contributions of the perturbations dominate. In the opposite regime, for small frequencies, the acoustic waves scatter weakly, which implies that, for  $\omega \rightarrow 0$ , the field remains essentially non-interacting. At this point, one may wonder where the correlation length  $l_d$  fits in this whole scenario. From Eq. (17), we see that, for  $\Lambda \rightarrow \infty$  the loop integral is ultraviolet divergent. So, introducing correlations between the classical random fluctuations  $\nu(z)$  leads to regularized outcomes. Therefore a physically motivated way to obtain a non-divergent result is to use colored noise rather than a white noise. Another point is that in principle one could think of two different ways of performing the joint quantum and noise averages. In a static disordered medium one could first perform the average over quantum fluctuations for a given frozen disorder configuration and then average over the disorder configurations [17]. This would correspond in random magnetic systems to the case of a quenched disorder. In the fluctuating medium considered in the present paper, the quantum and disordered averages cannot be separated and must be performed simultaneously – see e.g. Ref. [18]. This case is exactly the case of annealed systems, where the impurity degrees of freedom are in equilibrium with the others degrees of freedom of the system.

Coming back to fluid dynamics, the density-density correlation function can be calculated from the relation  $\psi(t, z) = u^2(z)\delta\rho(t, z)$ . From Eqs. (9) and (14), one has

$$\begin{aligned} \delta\rho(\omega, z) = & \delta\rho_0(\omega, z) + \nu(z)\delta\rho_0(\omega, z) + \sum_{n=1}^{\infty} (-1)^n \left( \frac{\omega}{u_0} \right)^{2n} \int \left[ \prod_{m=1}^n dz_m D_F(\omega, z - z_m) \nu(z_m) + \right. \\ & \left. + \nu(z) \int \prod_{m=1}^n dz_m D_F(\omega, z - z_m) \nu(z_m) \right] \delta\rho_0(\omega, z_n), \end{aligned} \quad (18)$$

where  $\delta\rho_0(\omega, z) = \psi_0(\omega, z)/u_0^2$  and we used  $D_F$  for the

$\delta\rho(t, z)$  field since its Green's function obeys the same

equation as  $G_F$  for the  $\psi(t, z)$  field. Taking the noise and quantum averages, the correlation function up to second order in  $\nu(z)$  is given in Fourier space by

$$\langle\langle\delta\rho(\omega, k)\delta\rho(\omega', k')\rangle\rangle_\nu = (2\pi)^2\delta(\omega - \omega')\delta(k - k')S_\rho(\omega, k), \quad (19)$$

with

$$S_\rho(\omega, k) = S_\psi(\omega, k) + \sigma^2 \int dq \left[ iD_F(\omega, q) - 4i\frac{\omega^2}{u_0^2}D_F(\omega, k)D_F(\omega, q) \right] h_\Lambda(k - q). \quad (20)$$

Note that the induced interactions are different for the  $\delta\rho$  field from those for the  $\psi$  field. This is due to the wave equation for  $\delta\rho$  is not of the form of Eq. (6), since on replacing  $\psi(t, z) = u^2(z)\delta\rho(t, z)$  in this equation there are extra spatial derivatives of  $u(z)$ . Therefore, although similar, the model here is different from the ones studied in condensed matter physics [14]. As before, the colored noise acts as a regulator of the loop integrals. Of course, there is a natural cut-off in the high frequency regime, not related to  $\Lambda$ , in the situation where the wavelength of the acoustic phonon is of the order of the interatomic separation  $l_i$ , i.e.,  $\omega^{-1} \approx l_i$ . For  $\omega^{-1} < l_i$  the field theoretical description of the acoustic excitation is no longer valid.

In summary, we have constructed a condensed matter analog model for fluctuations of the geometry of space-time due to quantum gravity effects. The model leads to the interpretation that such classical background fluctuations induce effective interactions on free fields. We expect that such sound cone fluctuations can be tested in suspensions like colloids since excitations of acoustic modes in these environments are described by random wave equations as the one we discussed in this paper [19]. A possible course of action is to remember the reference [14], where it is shown that a non-linear dispersion relation generated by a random wave equation induces peaks in the density of states. Another direction is the analysis of localization of acoustic waves. Such issue is related to the problem of how sound cone fluctuations in our analog model can change the variation of flight of pulses. The study of wave localization using diagrammatic perturbation method was presented in the Refs. [20–23]. It is known that in one dimension any disorder is strong enough to induce exponential localization of eigenmodes. On the other hand, a random fluid with a supersonic acoustic flow is also an analog model that opens the possibility to discuss the effect of the fluctuation of the geometry in the Hawking radiation. Specific examples studying Anderson localization in the analog model of the present paper as well as the implementation of randomness in an acoustic black hole will be reported elsewhere.

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